Obviously, any function which can be approximated by a function of this form can be inverted on a computing machine. A program to evaluate the constants in such an empirical equation can also be written. Thus the entire process of empirical representation, evaluation of singularities, and inversion can be carried out formally by machine. Since the use of the empirical representation is not rigorous, the solution would have to be tested.

CONCLUSIONS

Complex Laplace transforms can be inverted approximately by a number of simple techniques. Although the validity of the techniques is not easy to establish rigorously, the results can usually be tested by physical reasoning. By the use of empirical representations, almost any function can be inverted on a computing machine. These techniques extend the usefulness of operational calculus to many complex problems in chemical engineering.

ACKNOWLEDGMENT

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NOTATION

c = heat capacity
C_0 = original concentration of component 1
$C_n(t)$ = concentration of nth component
$c(s)$ = transformed concentration of nth component
$erfc\,(x)$ = complementary error function of x
h = heat transfer coefficient for convection
h_r = heat transfer coefficient for radiation
$I_0(x)$ = $J_0(ix)$
$J_0(x)$ = Bessel function of first kind and zero order of x
$J_1(x)$ = Bessel function of first kind and first order of x
$K_{1/4}(x)$ = modified Bessel function of second kind and one-quarter order of x
k = thermal conductivity
k_n = rate constant for nth reaction
L = dimensionless distance between spheres
r = fraction of radius
s = variable of transformation
t = time
T = temperature or dimensionless temperature
T_s = temperature of surroundings
$u(s)$ = transformed temperature or dimensionless temperature
z = distance from surface
ϕ = dimensionless parameter
ρ = density

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Fluidization and Sedimentation of Spherical Particles

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Data are presented in support of an expression describing the relation between the sedimenting velocity or the fluidizing velocity and the fraction voids. This expression which contains no empirical constants may be obtained by considering a particle in a fluid having the average properties of the suspension. Stokes's law is used to calculate the force on the particle, and an equation derived by Vand is used to describe the viscosity of the suspension. The equation based on this model is valid for particle Reynolds numbers less than 0.07. The model may be used as an approximation of bed behavior at higher Reynolds numbers by application of a correction to Stokes's law.

The steady state settling rate of a single particle in a fluid or the fluid velocity necessary to suspend a single particle may be described by equating the force of gravity to the viscous drag of the fluid. For spherical particles

Drag force on particle

$$= C_D \tfrac{1}{2} \rho U_0^2 \left(\frac{\pi D_p^2}{4}\right) \qquad (1)$$

Force of gravity

$$= (\rho_s - \rho) g \left(\frac{\pi D_p^3}{6}\right) \qquad (2)$$

$$U_0 = \sqrt{\frac{4gD_p(\rho_s - \rho)}{3\rho C_D}} \qquad (3)$$

For low settling rates ($Re_p < 0.1$) the drag coefficient C_D may be described by Stokes's law, and Equation (3) becomes

$$U_0 = \frac{(\rho_s - \rho) g D_p^2}{18\mu} \qquad (4)$$

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At high settling rates experimental measurements of C_D reported in the literature may be used. Equations (3) and (4) are not valid to predict the rate of settling of a bed of particles or the velocity necessary to suspend a bed of particles at a given voidage. The surrounding particles affect the flow field, and therefore the experimental conditions under which C_D is determined for single particles are not reproduced.

A number of derivations have been presented in the literature to describe the settling velocity or the fluidization velocity of beds of particles. Most of these have been empirical. Two derivations, which contain no empirical constants, describe the behavior of fluidizing or sedimenting beds at low Reynolds numbers. Brinkman (*2*), who calculated the force on a particle embedded in a porous mass, described the flow through the mass by Darcy's equation. The following equation for the sedimenting velocity is obtained if the viscosity term in his derivation is assumed to be that of the pure liquid:

$$\frac{U_c}{U_0} = 1 + \frac{3}{4}c\left(1 - \sqrt{\frac{8}{c} - 3}\right) \qquad (5)$$

Hawksley (*3*) applied a derivation presented by Vand (*4*) for the viscosity of concentrated suspensions of spherical particles:

$$\mu_c = \mu \exp\left(2.5c/1 - \frac{39}{64}c\right) \qquad (6)$$

He describes the force on each of the particles in the system with Stokes's law. However the properties of the fluid are altered to account for the presence of the other particles. The viscosity is given by Equation (6), and the density is that of the suspension

$$\rho_c = c\rho_s + (1 - c)\rho \qquad (7)$$

The relative velocity between the fluid and the particles of the suspension is equal to the settling velocity divided by the void fraction:

$$U = \frac{U_c}{1 - c} \qquad (8)$$

Hawksley substituted for U_0, ρ, and μ in Equation (4) the expressions given by Equations (6), (7), and (8).

$$U_c = \frac{(\rho_s - \rho) g D_p^2 (1 - c)^2}{18\mu \exp(2.5c/1 - \frac{39}{64}c)} \qquad (9)$$

$$\frac{U_c}{U_0} = (1 - c)^2 \exp(-2.5c/1 - \tfrac{39}{64}c)$$

Equations (5) and (9) are plotted in Figure 1. It can be seen that the results of the derivation of Brinkman and of that of Vand and Hawksley are quite different. Data have been presented in the literature to support both theories (*3, 5*).

The experimental investigation reported in this paper was undertaken to examine the validity of these two models. Measurements of the effect of void fraction upon the settling rate and the fluidization velocity were made for beds of spherical particles. By varying the viscosity of the fluid, experiments could be conducted over a large range of Re_p.

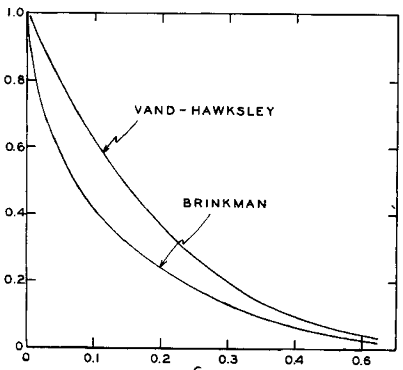


Fig. 1. Comparison of derivations of Brinkman and Hawksley.

DESCRIPTION OF EXPERIMENTS

The fluidization and sedimentation of 0.022-in. steel spheres (ρ_s = 7.43 g./cc.) and of 0.028-in. glass spheres (ρ_s = 2.88 g./cc.) in glycerine-water solutions were studied. The bed was contained in a 4-in.-diam. glass column and the liquor was circulated through the column from a holding tank, as indicated in Figure 2.

The behavior of the bed was dependent on the type of calming section employed.

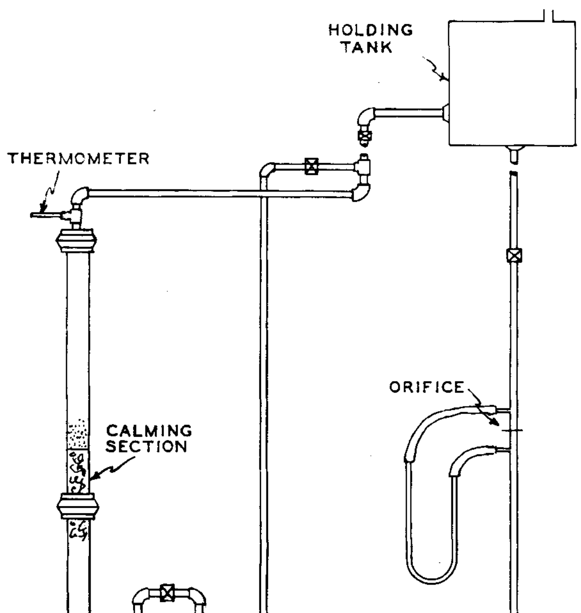


Fig. 2. Equipment used in experiments.

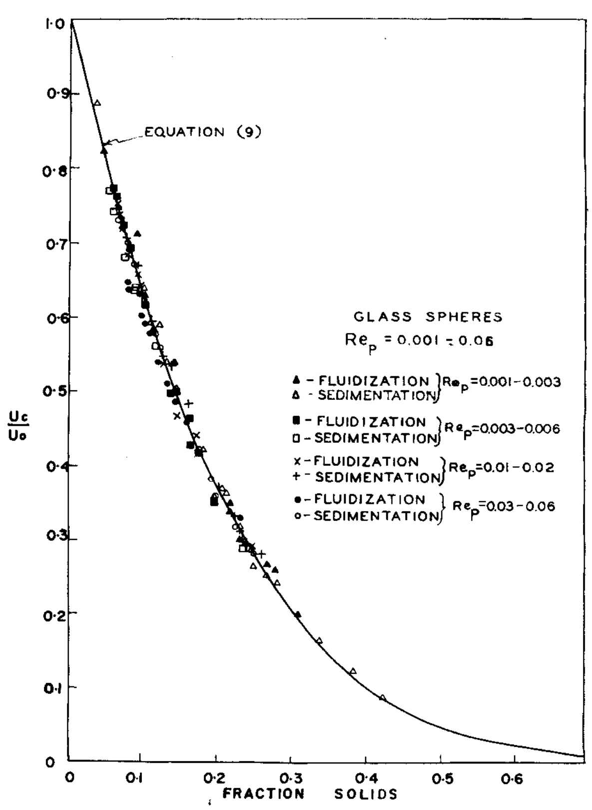


Fig. 3. Fluidization and sedimentation data for glass spheres; Re_p = 0.001 − 0.06.

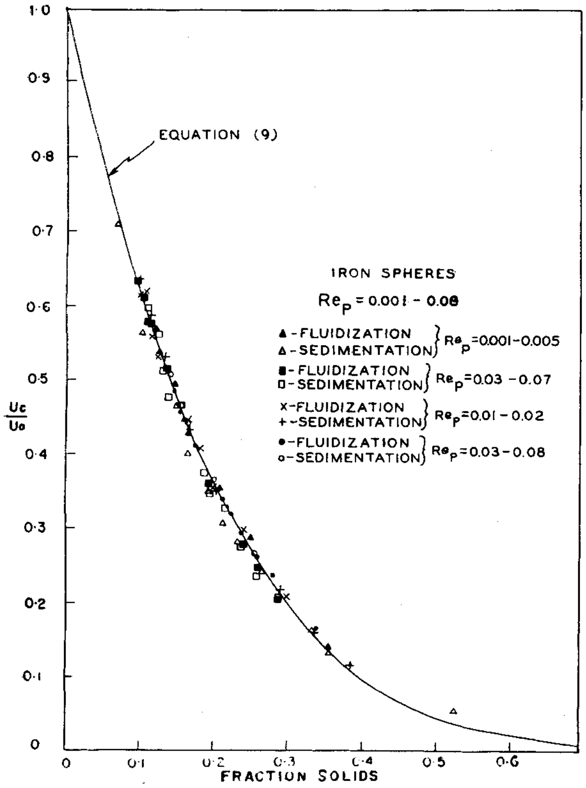


Fig. 4. ↑ Fluidization and sedimentation data for iron spheres; Re_p = 0.008 − 0.07.

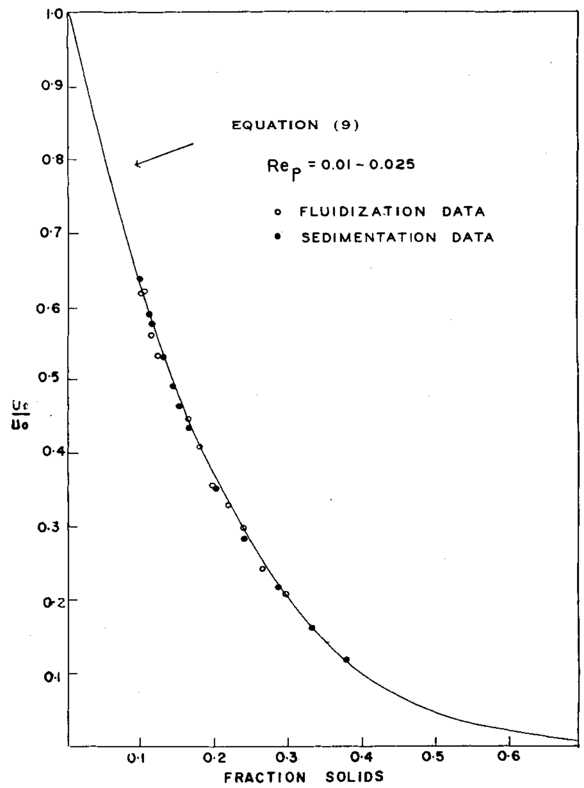
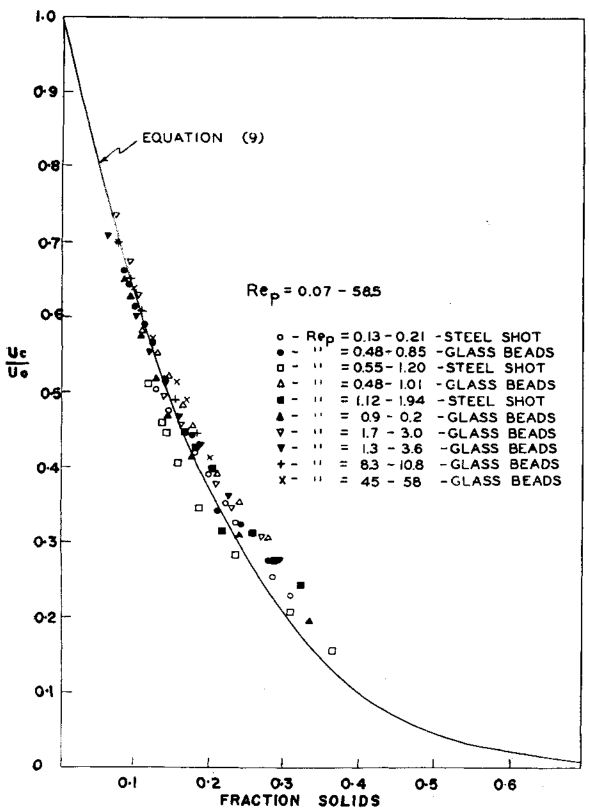


Fig. 5. Comparison of sedimentation and fluidization data for a typical run. ↑



← Fig. 6. Fluidization and sedimentation data at high Reynolds numbers; Re_p = 0.07 to 58.5.

A uniform flow into the bed was necessary in order to obtain meaningful results. This was especially true at high void fractions. In order to obtain uniform flow a 7-in. section of copper turnings was introduced below the bed support and metal baffles were placed in the elbow. The bed support consisted of a copper screen soldered to a thin metal plate which was shaped to fit snugly in the column. A gravity feed was employed for low flow rates, and at high throughputs a centrifugal pump was used. As fluids of high viscosity heated upon being circulated through the system, a heat exchanger controlled the temperature of the inlet fluid before it reached the column.

Special precautions were undertaken to obtain uniform spherical particles. The glass beads were manufactured by the Minnesota Mining and Manufacturing Company and the steel shot by the Wheelabrator and Equipment Corporation. Particles retained between Tyler sieves 20 and 35 were screened through specially designed equipment. A number of highly polished soft-steel slabs were supported on two threaded screws. By means of positioning and locking nuts the gaps between the slabs could be accurately adjusted, and the particles were sieved through these openings. After the sieving the particles were rolled down an 18-in. incline of 10°. Nonspherical particles stuck to the incline and were discarded. The process was repeated with an incline of 5°.

Data were taken over a viscosity range of 1 to 390 centipoise and a density range of 1.00 g. to 1.25 g./cc. The particle Reynolds number Re_p varied between 0.001 and 58.2; the Reynolds number of the liquid in the empty column before entry into the bed varied from 0.12 to 7,700. Fluidization data were taken over the entire range, whereas it was possible to perform sedimentation experiments only at the lower Reynolds numbers.

RESULTS

Data obtained at particle Reynolds numbers less than 0.07 agreed with the Vand-Hawksley theory. In Figures 3 and 4 the results for low Reynolds numbers are presented as a plot of U_c/U_0 vs. fraction solids. In fluidization runs U_c is the fluid velocity based on the empty tube. In sedimentation runs U_c is the rate at which the bed settles. The solid line in Figures 3 and 4 is Equation (9).

In runs where both fluidization and sedimentation data were obtained no difference could be noted, even at higher

Reynolds numbers. Data from a typical run are shown in Figure 5.

Data at higher Reynolds numbers are presented in Figure 6, the terminal velocity U_0 being calculated by the use of Equation (3). Data from reference *6* were used to evaluate C_D, and the Reynolds number used was $Re_p = (D_p U_c \rho)/(1 - c)\mu)$. The curve drawn in Figure 6 was calculated from Equation (9). The trend of individual runs did not agree too well with the Vand-Hawksley theory; however, the theory appears to be an approximation of the data obtained in this research at Reynolds numbers bebetween 0.07 and 58.5.

A complete tabulation of the data and results of this research is contained in reference *1*.

VISUAL OBSERVATIONS

At particle Reynolds numbers below 0.8 the bed had a uniform appearance, with no large mass movements of the particles. The particles, however, were continually coming together in small groupings and then dispersing. At particle Reynolds numbers above 2 there became evident mass movements of groups of particles. At the highest Reynolds numbers for both the iron shot and the glass spheres the particle flow pattern consisted of a random eddying motion. There were large variations in the solids concentration, and the motion was quite similar to that obtained in gas-solid systems.

NOTATION

c = fraction solids
C_D = particle drag coefficient
D_p = particle diameter
g = acceleration of gravity
Re_p = particle Reynolds number = $D_p U_c \rho/(1 - c)\mu$
U_0 = free fall velocity of a single particle
U_c = settling velocity of a sedimenting bed; velocity of the fluid based on the empty tube cross section
U = actual fluid velocity
μ = fluid viscosity
μ_c = viscosity of a suspension
ρ = fluid density
ρ_s = solid density
ρ_c = density of a suspension

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